TECHNICAL NOTE

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The Application of Linear Regression to Range-of-Fire Estimates Based on the Spread of Shotgun Pellet Patterns

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ABSTRACT: A series of shotgun pellet patterns were fired at ranges that varied from 3.9 to 15.2 m (10 to 50 ft) using a 12-gauge cylinder-bored shotgun firing No. 2 chilled shot cartridges. The spreads of the pellet patterns were measured in two ways: the radius of the smallest circle that would enclose the entire pattern was measured with a transparent overlay and the square root of the area of the smallest rectangle that would enclose the pattern was calculated. Linear regression analysis was applied to sets of data for shots at three, five, and nine ranges. For both measures of pattern spread the linear regression gave correlation coefficients greater than 0.99, indicating that a linear relationship existed between the measures of the pattern spreads and the range of fire. The confidence intervals for range-of-fire estimates at the 95% confidence level were calculated for each set of data. As expected, increasing the number of test-fired patterns decreases the confidence interval and so improves the range-of-fire estimates obtained from the test-fired patterns.

KEYWORDS: criminalistics, ballistics, shotguns

Firearms examiners are occassionally asked to estimate the range from which a shotgun was fired into a target (commonly a human being). The examiner must test-fire a number of pellet patterns using the suspect shotgun and ammunition from the same batch as that used to fire the questioned pattern, if available. Examiners may compare the test-fired patterns directly with the questioned pattern whose range of fire is to be estimated. The range-of-fire estimate is then based on impressions of overall size of pattern and the pellet density [*1*]. Alternately, the sizes of the questioned pattern and of the test-fired patterns may be measured by some method [*2-4*]. Although the resulting measurements may be treated by the methods of regression analysis, discussions of range-of-fire estimates obtained by regression analysis have not appeared in the forensic science literature. This is unfortunate since regression analysis provides a basis for determining the precision of range-of-fire estimates and hence their probative value. The

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availability of programmable pocket calculators and minicomputers with sophisticated statistical packages place an important and powerful data reduction tool in the hands of firearms examiners. We report here on the application of linear regression to a set of test-fired shotgun pellet patterns. With these data we will demonstrate the calculation of confidence intervals for range-of-fire estimates and the effect of increasing the number of test-fired patterns.

Experimental Procedure

A Remington Model 870 12-gauge shotgun with a 66-mm (26-in.) cylinder-bored barrel was used to fire Remington 12-gauge 70-mm (2¾-in.) 42.6-g (1½-oz) express magnum chilled No. 2 shot cartridges (nominal pellet diameter 3.8 mm [0.15 in.]; 135 pellets per round) at 889- by 1145-mm (35- by 45-in.) anthropomorphic silhouette targets. The ranges of fire varied from 3.9 to 15.2 m (10 to 50 ft) in 1.5-m (5-ft) increments. One shot was fired at each range for a total of nine rounds.

The spread of each pattern was determined in two ways. The area A of the smallest circumscribed rectangle that would enclose the pellet pattern was obtained as the product of the largest horizontal and vertical dimensions of the pellet patterns. As the shot string travels toward the target, it spreads out in both the horizontal and vertical directions. The area A should therefore be a quadratic function of the range. In order to avoid the additional computations required for a nonlinear regression, the square root of A was used in our regression analysis.

A 12.2-cm (40-in.) plastic overlay marked with concentric circles was placed over each pellet pattern and moved about until its center appeared to coincide with the center of the pellet pattern. The radius R of the smallest circle that would completely enclose the pellet pattern was then read from the overlay. We did not attempt to exclude so-called "flyers," pellets that appear to be scattered outside the pattern area proper. We did not feel that valid statistical reasons existed for such exclusion. As will be seen, inclusion of the "flyers" does not appear to affect adversely the linear regressions.

Regression Analysis

Linear regressions were performed on the data obtained from the test-fired pellet patterns. The model used was

$$y = a + bx$$

where x is the range, y is $\sqrt{A}$ or R, a is the intercept, and b is the slope. The values a and b were calculated by standard methods [5]. In addition, the correlation coefficient r and the standard error S_e were calculated.

Three sets of data were analyzed by linear regression. The first set contained $\sqrt{A}$ and R for three ranges: 3.0 m (10 ft), 9.1 m (30 ft), and 15.2 m (50 ft). The second set contained $\sqrt{A}$ and R for five ranges: 3.0 m (10 ft), 6.1 m (20 ft), 9.1 m (30 ft), 12.2 m (40 ft), and 15.2 (50 ft). The final set contained $\sqrt{A}$ and R for all nine shots.

Results and Discussion

The results of the linear regressions are shown in Table 1. All the correlation coefficients are close to one, indicating that linear functions adequately describe the relationships between the range of fire and either $\sqrt{A}$ or R, at least over the ranges covered by the test shots. The negative intercepts calculated in all cases require that $\sqrt{A}$ and R become zero at a point between 1.2 and 1.8 m (4 and 6 ft) in front of the muzzle of the shotgun. This result is consistent with the observation of Berg [6] that shotgun pellets travel together for some distance before they begin to spread out.

TABLE 1—*Results of the linear regressions.*[a]

Measure of Pattern Spread	Number of Patterns k	Regression Intercept a, in.	Coefficient Slope b, in./ft	Correlation Coefficient r	Standard Error of Estimate S_e, ft	$(tS_e/\|b\|)\cdot\sqrt{1+(1/k)}$, ft[b]
Radius of smallest circumscribed circle R	3	−1.703	0.342	0.999	0.536	23.5
Square root of area of smallest circumscribed rectangle $\sqrt{A}$	3	−3.687	0.626	0.993	2.153	49.9
R	5	−1.663	0.348	0.997	0.602	5.9
$\sqrt{A}$	5	−3.333	0.620	0.994	1.078	5.9
R	9	−1.340	0.336	0.995	0.475	3.5
$\sqrt{A}$	9	−2.999	0.620	0.994	0.735	3.0

[a] 1 in. = 25.4 mm and 1 ft = 0.3048 m.
[b] t chosen for a confidence level of 95%.

Suppose we wish to use the regression lines to estimate the range from which a given pellet pattern was fired. The estimated range $\hat{x}$ and its confidence interval are given by [5]:

$$\hat{x} = (\bar{y}' - a)/b \pm (tS_e/|b|)\sqrt{(1/m) + (1/k) + (\bar{y}' - a - b\bar{x})^2/[b^2\Sigma(x_i - \bar{x})^2]}$$

where $\bar{y}'$ is the mean of either $\sqrt{A}$ or R for m pellet patterns fired at the same range, x_i is the range at which test patterns were fired to determine the regression line, $\bar{x}$ is the mean of x_i, t is Student's t, and k is the number of pellet patterns. Normally, $\bar{y}'$ will be the value of $\sqrt{A}$ or R for a single pellet pattern and m will be one.

To show the effect of increasing the number of the pellet patterns fired to determine the regression line, we choose to let

$$\bar{y}' = a + b\bar{x}$$

Such a choice yields the narrowest confidence interval for any particular confidence level. If $m = 1$, the confidence interval expression above then reduces to

$$(tS_e/|b|)\sqrt{1 + (1/k)}$$

This term is included in Table 1, calculated for a confidence level of 95%. As may be seen, increasing k from three to nine produces a dramatic decrease in the width of the confidence interval. Additional test-fired patterns would produce an additional narrowing of the confidence interval; however, examiners in practice must balance the demand for high precision with a realistic assessment of the effort required to attain it.

There is no significant difference in the use of R or $\sqrt{A}$ as far as the confidence intervals of the range estimates are concerned. The examiner should use the measure of pellet pattern spread that he finds most convenient.

Several precautions should be noted. The firearms examiner should not assume a priori that a linear function adequately describes the relationship between the measure of the spread of the pellet pattern being used and the range of fire. This is a hypothesis that must be tested by examining the correlation coefficient. The examiner must also decide on the confidence level for which the confidence interval is to be calculated. The higher the confidence level desired the wider the resulting confidence interval [5]. Finally, no regression analysis can produce a valid range estimate unless the test-firings duplicate the conditions under which the questioned pattern was produced: this means using the same weapon and ammunition from the same batch.

Conclusion

The method of linear regression was applied to a set of shotgun pellet patterns fired from different ranges. The spread of each pattern was obtained in two ways: by finding the radius of the smallest circle that would enclose just the pattern and by computing the square root of the area of the pattern. These measures of the spreads of the pellet patterns were then subjected to linear regressions. The linear model for the relationship between the range of fire and both measures of the spread of the pellet patterns was found to fit the data well. As expected, the confidence limits for the range-of-fire estimates based on these data were found to improve as the number of test-fired shots increases.

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